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Reg. No. : ....

Name : ....

Third Semester B.Tech. Degree Examination, November 2014
(2013 Scheme)
13.301 : ENGINEERING MATHEMATICS – II
(ABCEFHMNPRSTU)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. If $\vec{r}$ is a vector of constant magnitude, show that $\vec{r}.\frac{d\vec{r}}{dt} = 0$.

2. Obtain the Fourier series of x cos x in $-\pi < x < \pi$.

3. Show that $F[f(ax)] = \frac{1}{a}F\left(s/a\right)$ if F [f(x)] = F(s).

4. Obtain the partial differential equation by eliminating the arbitrary function 'F' from $F\left(\frac{z}{x+y}\right) = xy + yz + xz$.

5. Solve by the method of separation of variables $u(x, t) = e^{-t}\cos x$ with $u(x, 0) = 0$ and $\frac{\partial u}{\partial t}(0, t) = 0$.

PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

Module – I

6. a) Find the angle between the directions of the velocity and acceleration vectors at time 't' of a particle with position vector $\vec{r} = (t^2 + 1)\hat{i} - 2t\hat{j} + (t^2 - 1)\hat{k}$.

P.T.O.

b) Evaluate $\int_C \vec{F} \times d\vec{r}$ along the curve $x = \cos\theta$, $y = \sin\theta$, $z = 2\cos\theta$ from $\theta = 0$ to $\theta = \pi/2$, given that $\bar{F} = 2y\hat{i} - z\hat{j} + x\hat{k}$.

c) Evaluate $\iint_S (yz^2\hat{i} + xz^2\hat{j} + 2z^2\hat{k}).d\bar{s}$, where s is the closed surface bounded by the xoy-plane and the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ above this plane.

7. a) Find the value of 'n' if $r^n \bar{r}$ is both solenoidal and irrotational.

b) Using Green's theorem in a plane evaluate $\int_c (3x^2 - 8y^2)\,dx + (4y - 6xy)dy$ where 'c' is the boundary of the region defined by the lines x = 0, y = 0, x+y = 1.

c) Verify Stoke's theorem when $\bar{F} = (2xy - x^2)\hat{i} - (x^2 - y^2)\hat{j}$ and c is the boundary of the region enclosed by the parabolas $y^2 = x$ and $x^2 = y$.

Module – II

8. a) Find the Fourier series expansion of the function.

$$F(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 \le x \le 2 \end{cases}$$

b) Using Fourier integral show that $\int_0^\infty \frac{\lambda \sin \lambda x}{\lambda^2 + k^2}\, d\lambda = \frac{\pi}{2} e^{-kx}$ $x > 0$, $k > 0$

9. a) Find the Fourier series of

$$F(x) = \begin{cases} 1, & -2 < x < 0 \\ e^{-x}, & 0 < x < 2 \end{cases}$$

b) Find the Fourier cosine transform of $F(x) = e^{-4x}$ and hence deduce that $\int_0^\infty \frac{\cos 2x}{x^2 + 16}\, dx = \frac{\pi}{2} e^{-8}$.

Module – III

10. a) Solve the partial differential equation $p(1+q) = qz$.

b) Solve the partial differential equation $z(z^2 + xy)(px - qy) = x^4$.

c) Solve the partial differential equation $(D^2 + D'^2) z = x^2y^2$.

11. a) Solve the partial differential eqūation by Charpit's method $z^2 = pqxy$.

b) Solve the partial differential equation

$$(D^2 - 6DD' + 9D'^2)\ z = e^{3x+4y} + \sin(x+y).$$

Module – IV

12. a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in the position $y(x, 0) = k(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement function $y(x, t)$.

b) The ends A and B of a rod 10 cm long have the temperatures at 20°C and 40°C until steady-state prevails. The temperature of the ends are changed to 50°C and 10°C respectively. Find the subsequent temperature function $u(x, t)$ at time 't'.

13. a) Find the displacement of a string stretched between two fixed points at a distance '2c' apart when the string is initially at rest in equilibrium position and points of the string are given initial velocities 'V' where

$$V = \begin{cases} x/c, & 0 < x < c \\ \dfrac{2c - x}{c}, & c < x < 2c. \end{cases}$$

b) The equation for the conduction of heat along a bar of length 'l' is $\dfrac{\partial u}{\partial t} = \alpha^2 \dfrac{\partial^2 u}{\partial x^2}$. Find an expression for u, if the ends of the bar are maintained at zero temperature and if initially the temperature is T at the centre of the bar and falls uniformly to zero at its ends.
